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Curvature Perturbations and the Curvaton

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Work in progress with

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The Cult of ζ

- Why ζ:
  - Very convenient for comparing models with observations (sources LSS)
  - Nice independent variable for evolution equations
- Curvature perturbation on uniform density hypersurfaces

$$\zeta = -\psi - H\frac{\delta\rho}{\dot{\rho}}$$

Bardeen 1980
Bardeen, Steinhardt and Turner 1983

where:
ψ: curvature perturbation
H: Hubble parameter
ρ and $\delta\rho$: background and perturbed energy density

- Multi-fluid case: curvature perturbation on uniform α-fluid density hypersurfaces

$$\zeta_\alpha = -\psi - H\frac{\delta\rho_\alpha}{\dot{\rho}_\alpha}$$

- Relation between ζ_α and total ζ

$$\zeta = \sum_\alpha \frac{\dot{\rho}_\alpha}{\dot{\rho}}\zeta_\alpha$$

- Isocurvature or relative entropy perturbation

$$S_{\alpha\beta} = \zeta_\alpha - \zeta_\beta$$

Evolution

Time evolution of the curvature perturbation on large scales for multiple fluids including energy transfer

$$\dot{\zeta} \simeq \frac{1}{\rho + P} \sum_{\alpha} \left\{ \dot{\rho}_{\alpha} c_{\alpha}^{2} \sum_{\beta} \frac{\dot{\rho}_{\beta}}{\dot{\rho}} S_{\alpha\beta} - H \delta P_{\text{intr},\alpha} \right\}$$

M., Wands, and Ungarelli 2002

Here the intrinsic entropy or non-adiabatic pressure perturbation of α-fluid is

$$\delta P_{\text{intr},\alpha} = \delta P_{\alpha} - c_{\alpha}^{2} \delta \rho_{\alpha}$$

where:
ρ, ρ_α: densities, $\delta\rho$, $\delta\rho_\alpha$: perturbed densities
P: pressure, δP, δP_α: perturbed pressure
adiabatic sound speed of the α-fluid: $c_\alpha^2 \equiv \dot{P}_\alpha / \dot{\rho}_\alpha$

- $\dot{\zeta} \simeq 0$ if

  * no relative entropy perturbation,
    i.e. $S_{\alpha\beta} = 0 \Rightarrow \zeta_\alpha = \zeta_\beta$ (curvature perturbations of all fluids must be equal)

  * no intrinsic entropy perturbation, $\delta P_{\text{intr}} = 0$:

  $\Rightarrow$ Total curvature perturbation is constant on large scales for purely adiabatic perturbations.

- To generate ζ need:

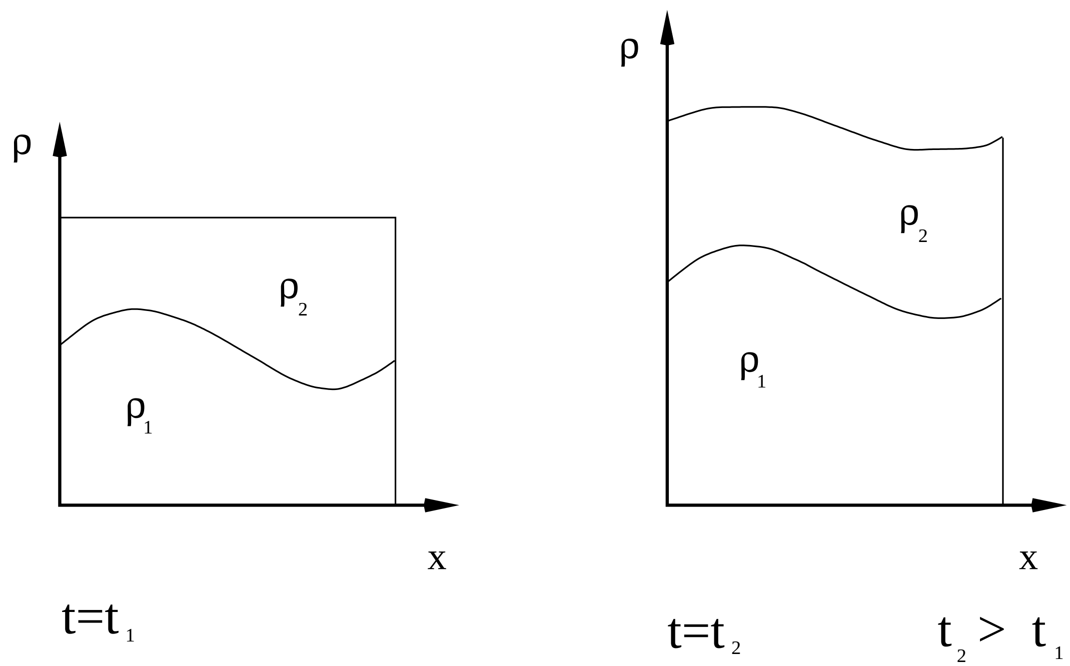
  – non-zero intrinsic entropy perturbation
    $\delta P_{\text{intr}} \neq 0$, and/or

  – non-zero isocurvature or relative entropy perturbations $S_{\alpha\beta} \neq 0$.
    $\Rightarrow$ The Curvaton

Perturbation mutations

Isocurvature perturbations can generate density perturbations

Mollerach 1990
Lyth and Wands 2001



- Initially: no density perturbation, i.e. only isocurvature perturbations

- Due to *different evolution* of the fluids, get density perturbation at some later time

Evolution of Entropy

Evolution equation for $S_{\alpha\beta}$ on large scales including energy transfer

$$\dot{S}_{\alpha\beta} \simeq H\left(\frac{3H\delta P_{\mathrm{intr},\alpha} - \delta Q_{\mathrm{intr},\alpha}}{\dot{\rho}_\alpha} - \frac{3H\delta P_{\mathrm{intr},\beta} - \delta Q_{\mathrm{intr},\beta}}{\dot{\rho}_\beta}\right) + \sum_\gamma \frac{\dot{\rho}_\gamma}{2\rho}\left(\frac{Q_\alpha}{\dot{\rho}_\alpha}S_{\alpha\gamma} - \frac{Q_\beta}{\dot{\rho}_\beta}S_{\beta\gamma}\right)$$

M., Wands, and Ungarelli 2002

where

- intrinsic non-adiabatic pressure perturbation of α-fluid

$$\delta P_{\mathrm{intr},\alpha} = \delta P_\alpha - c_\alpha^2 \delta\rho_\alpha$$

- intrinsic non-adiabatic energy transfer perturbation of α-fluid

$$\delta Q_{\mathrm{intr},\alpha} \equiv \delta Q_\alpha - \frac{\dot{Q}_\alpha}{\dot{\rho}_\alpha}\delta\rho_\alpha$$

  Energy transfer: Q_α (background), δQ_α (perturbed)

- – Above definitions are gauge-invariant

  – $\delta P_{\mathrm{intr},\alpha} = 0$ if $P_\alpha = P_\alpha(\rho_\alpha)$ and $\delta Q_{\mathrm{intr},\alpha} = 0$ if $Q_\alpha = Q_\alpha(\rho_\alpha)$

Energy conservation in the background

$$\dot{\rho}_\alpha + 3H(\rho_\alpha + P_\alpha) = Q_\alpha$$

Kodama and Sasaki 1984

A simple application: the curvaton

- Background equations:

$$\begin{aligned}\dot\rho_\sigma &+ 3H\rho_\sigma = Q_\sigma\\ \dot\rho_\gamma &+ 4H\rho_\gamma = Q_\gamma\\ \dot\rho_{\gamma,\mathrm{old}} &+ 4H\rho_{\gamma,\mathrm{old}} = 0\end{aligned}$$

- Modeling the energy transfer: curvaton simply decays into radiation

$$\begin{aligned}Q_\sigma &= -\Gamma\rho_\sigma\\ Q_\gamma &= \Gamma\rho_\sigma\end{aligned}$$

ρ_σ: curvaton energy density, ρ_γ: radiation
Γ: decay rate of the curvaton into radiation

$$\begin{aligned}\delta Q_\sigma &= -\Gamma\delta\rho_\sigma\\ \delta Q_\gamma &= \Gamma\delta\rho_\sigma\end{aligned}$$

δQ_σ,δQ_γ: perturbed energy transfer

- With this ansatz for decay, $\delta P_{\mathrm{intr},\alpha} = 0$ and $\delta Q_{\mathrm{intr},\alpha} = 0$
- Evolution equation of total curvature perturbation

$$\dot\zeta = \frac{3H}{\dot\rho}\frac{\dot\rho_\sigma\dot\rho_\gamma}{\dot\rho}c_\gamma^2 S_{\sigma\gamma}$$

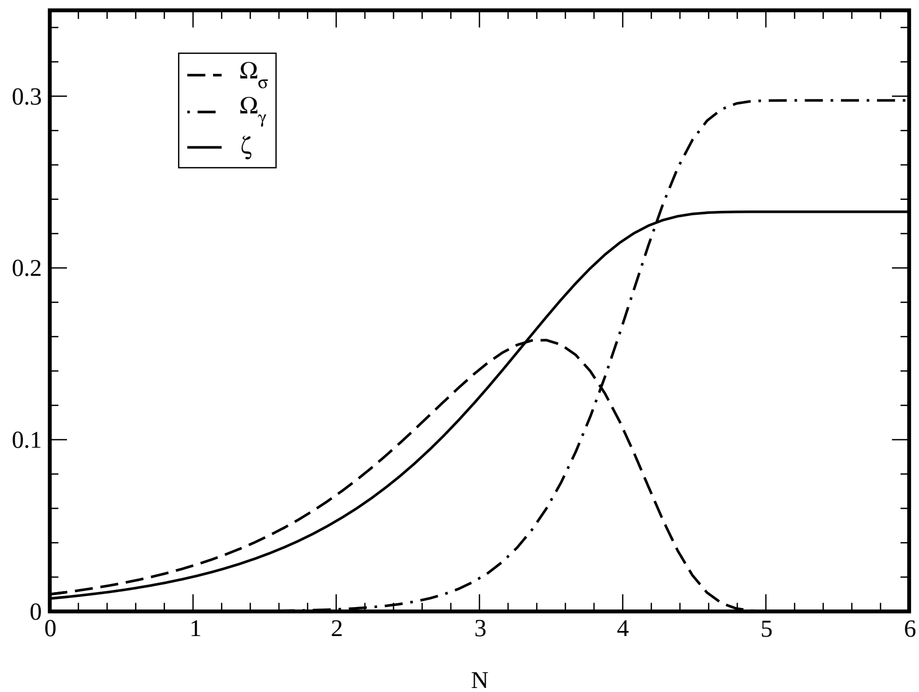
- Evolution equation for $S_{\sigma\gamma}$

$$\dot S_{\sigma\gamma} = \frac{\Gamma}{2}\frac{\dot\rho_\sigma}{\dot\rho_\gamma}\frac{\rho_\sigma}{\rho}\left(1 - \frac{\dot\rho_\gamma^2}{\dot\rho_\sigma^2} - 2\frac{\rho}{\rho_\sigma}\right)S_{\sigma\gamma}$$

where

$$S_{\sigma\gamma} = \zeta_\sigma - \zeta_\gamma\,,$$

Numerical Solution



$\Omega_\sigma \equiv \frac{\rho_\sigma}{\rho}$ and $\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho}$: normalised curvaton and radiation densities

$N \equiv \ln a$: e-foldings

Initial conditions and parameters:

$\Gamma = 0.001$

$\Omega_{\text{total}} = 1$
$\Omega_\sigma = 0.01$
$\Omega_\gamma = 0$
$\zeta_\sigma = 1$

Conclusions

- Evolution of curvature perturbation on large scales, for multiple and interacting fluids, is sourced only by non-adiabatic terms:

$$\dot{\zeta} \sim \sum_{\alpha\beta} S_{\alpha\beta}, \delta P_{\text{intr},\alpha}$$

- Evolution of entropy perturbations on large scales, for multiple and interacting fluids, is sourced only by non-adiabatic terms:

$$\dot{S}_{\alpha\beta} \sim \sum_{\alpha\beta} S_{\alpha\beta}, \delta P_{\text{intr},\alpha}$$

- Formalism provides for a convenient and unambiguous way to study the evolution of perturbations including energy transfer

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